Correction

I made a mistake in class on Monday and I want to make sure you see the correct version before starting your homework.

When solving the matrix equation

$$\boldsymbol{x}' = A\boldsymbol{x},$$

if A has repeated eigenvalues $\lambda_1 = \lambda_2 = \lambda$, we find all the linearly independent eigenvectors corresponding to λ by solving

$$(A - \lambda I)\boldsymbol{v} = \mathbf{0}.$$

If there is only one linearly independent eigenvector $\boldsymbol{v}$, then we must find another generalized eigenvector $\boldsymbol{u}$ by solving

$$(A - \lambda I)\boldsymbol{u} = \boldsymbol{v}.$$

So far, this is the same as what we did in class, but I said that our final solution was

$$\boldsymbol{x} = C_1\boldsymbol{v}te^{\lambda t} + C_2\boldsymbol{u}e^{\lambda t}.$$

This is **not correct**.

Instead, one independent solution is

$$x_2 = \boldsymbol{v}te^{\lambda t} + \boldsymbol{u}e^{\lambda t}$$

and the other independent solution is

$$x_1 = \boldsymbol{v}e^{\lambda t}.$$

Our final solution is therefore

$$\boldsymbol{x}(t) = C_1\boldsymbol{v}e^{\lambda t} + C_2\left(\boldsymbol{v}te^{\lambda t} + \boldsymbol{u}e^{\lambda t}\right).$$